Correction to Section 4.1

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Abstract

The leading definition in Section 4.1 is confusing, which makes it difficult to derive (4.5) and equations that follow. This note clarifies the starting point and presents the details of the derivation. The author thanks Harold Knight and Kshitija Despande for pointing out the inconsistencies.

1 Background

The development in Chapter 4.1 leading to equations (4.5), (4.6), (4.7), and (4.8) is vague and there are some errors. The following development should clarify the reasoning, which is important because the results are essential to the rest of the development in Chapter 4.

The following definitions were introduced in Chapter 1:

$$\psi(x + x_m; \varsigma) = \iint \widehat{\psi}(x_m; \kappa) \exp\{ikg(\kappa) |x - x_m|\} \times \exp\{i\kappa{\cdot}\varsigma\} \frac{d\kappa}{(2\pi)^2}, \tag{1}$$

where

$$kg(\kappa) = k\sqrt{1 - (\kappa/k)^2} \tag{2}$$

is used to denote the x component of the propagation vector $[kg(\kappa), \kappa]$, which has constant magnitude k. The diffraction integral (1) formall translates a freely propagating field from the plane at x_m to the plane at x.

2 Derivation

For a field that is propagating predominantly along the ray defined by the fixed propagation vector

$$\begin{aligned}\mathbf{k} &= k\,[\cos\theta, \sin\theta\cos\phi, \sin\theta\sin\phi] \\ &= [kg(\mathbf{k}), \mathbf{k}_T]\,,\end{aligned} \tag{3}$$

it is desirable to perform the diffraction computation in a coordinate system whose reference slides along a ray aligned with $\mathbf{k}$. The sliding origin of the continuously displaced coordinate system (CDSC) is located at the position

$$\mathbf{R}_0(x) = [x, \tan\theta \widehat{\mathbf{a}}_{k_T} x]\,. \tag{4}$$

Let $\psi_{\mathbf{k}}(x, \rho)$ represent the observable field in the CDSC. We need to define a new diffraction operator that propagates $\psi_{\mathbf{k}}(x, \rho)$ in the CDSC system. This is with two operations. First the angular spectrum is centered the reference direction as follows:

$$\begin{aligned}
&\psi(x, \varsigma) \exp\{-i\mathbf{k}_T\cdot\varsigma\} \\
&= \iint \widehat{\psi}(0; \kappa') \exp\{ikg(\kappa')x\} \\
&\times \exp\{i(\kappa' - \mathbf{k}_T)\cdot\varsigma\} \frac{d\kappa'}{(2\pi)^2} \\
&= \iint \widehat{\psi}(0; \kappa + \mathbf{k}_T) \exp\{ikg(\kappa + \mathbf{k}_T)x\} \\
&\times \exp\{i\kappa\cdot\varsigma\} \frac{d\kappa}{(2\pi)^2}
\end{aligned} \tag{5}$$

The next step compensates for the lateral displacement of $\mathbf{R}_0(x)$ as x changes. In effect, the structure appears to be entering the CDSC coordinate system from the opposite direction of the coordinate system shift. Thus,

$$\begin{aligned}
\psi_{\mathbf{k}}(x, \rho) = &\iint \widehat{\psi}(0; \kappa + \mathbf{k}_T) \exp\{i(kg(\kappa + \mathbf{k}_T) - \tan\theta \widehat{\mathbf{a}}_{k_T})x\} \\
&\times \exp\{i\kappa\cdot\rho\} \frac{d\kappa}{(2\pi)^2}.
\end{aligned} \tag{6}$$

The direct substitution of $\varsigma = \rho + \tan\theta \widehat{\mathbf{a}}_{k_T} x$, which was suggested in a previous note, gives the wrong sign for a shift of the reference coordinate system. Some support that negative sign is necessary can be seen from (4.25), which is repeated here:

$$\begin{aligned}
kg(\kappa + \mathbf{k}_T) = &\, k\cos\theta \\
&\times \left[1 - \left(1 - \left(\frac{\kappa}{k\cos\theta}\right)^2 - 2\frac{\tan\theta \widehat{\mathbf{a}}_{k_T}\cdot\kappa}{k\cos\theta}\right)^{1/2}\right] \\
&\simeq (\tan\theta)\,\widehat{\mathbf{a}}_{k_T}\cdot\kappa + \frac{\kappa^2 + \tan^2\theta\,(\widehat{\mathbf{a}}_{k_T}\cdot\kappa)^2}{2k\cos\theta}.
\end{aligned} \tag{7}$$

In the narrow-angle scatter limit the $(\tan\theta)\,\widehat{\mathbf{a}}_{k_T}\cdot\kappa$ terms cancel giving the correct parabolic equation form.