SA21B-2019: Ramifications of Intermediate-Scale Ionospheric Structure for Tomographic Reconstruction Using Two-Dimensional Simulation

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Abbreviated Abstract

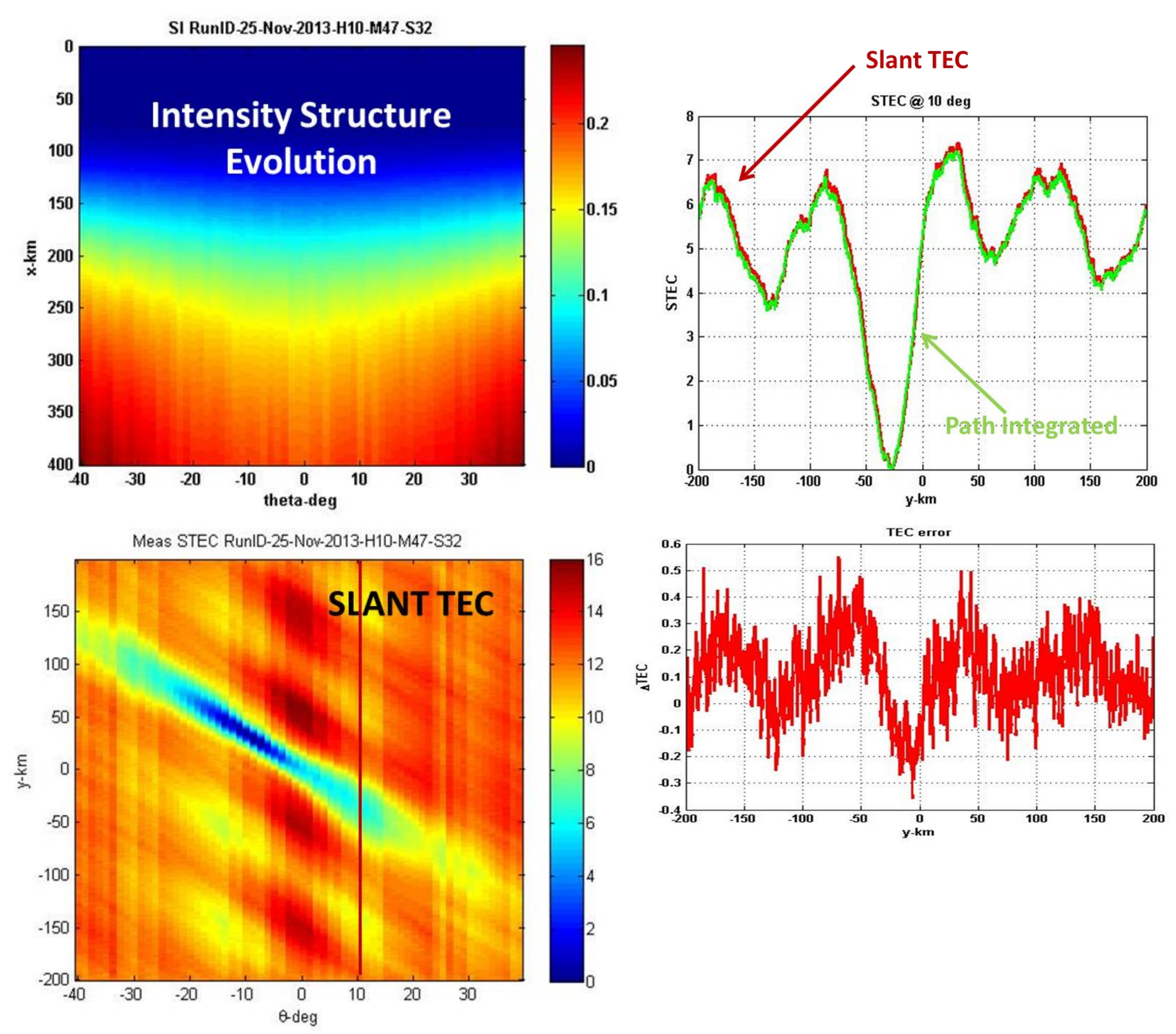
Global observation of the GPS satellite constellation for ionospheric diagnostics is now a worldwide activity driven by both practical and scientific objectives. Tomographic analysis is based on the assumption that the measured signal phase is proportional to path-integrated electron content (TEC). However, intermediate-scale structure causes a disruptive stochastic modulation of the GPS signals (scintillation). Scintillation is a nuisance for data assimilation, but the physical processes that generate intermediate-scale structure are intimately part of ionospheric physics.

A two-dimensional propagation model has been configured to explore the ramifications of intermediate-scale structure on TEC data assimilation and tomographic reconstruction. Although the propagation computation is essentially exact, meaningful conclusions depend on the fidelity of the intermediate scale structure mode. Models also play a role in tomographic reconstructions.

The interplay involves diffractive distortion of path-integrated phase, unresolved stochastic structure, and the geometric structure dependence imposed by extended magnetic-field-aligned structure.

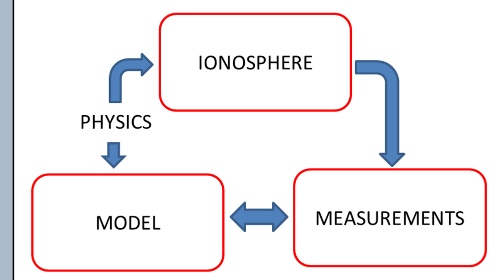
Full text can be found on http://chuckrino.com/wordpress/

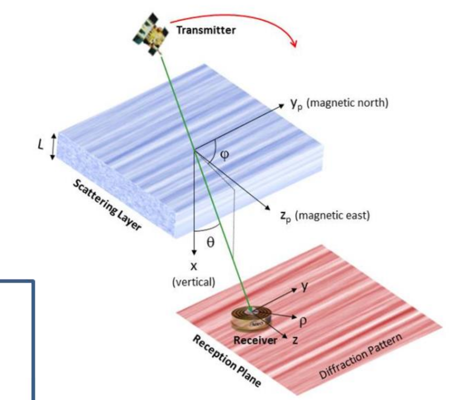
SLANT TEC Idealized



Summary & Conclusions

- Two-Dimensional simulations of propagation in extended media with large-scale and intermediate-scale structure were used to simulate slant TEC data for tomographic reconstruction
  - The tomographic ideal is generated by direct path integration
  - Reconstructed measurement-plane phase was compared to the tomographic ideal
- Preliminary results show that refraction under moderate scintillation conditions does impact the upper intermediate scale range (<100 km), which is the current resolution limit for tomographic reconstruction
- A direct tomographic inversion using modified SVD was demonstrated





STRUCTURE MODEL

$$n(\boldsymbol{\rho}) = \bar{n}(\boldsymbol{\rho})(1+\delta n(\boldsymbol{\rho})/n_0)$$

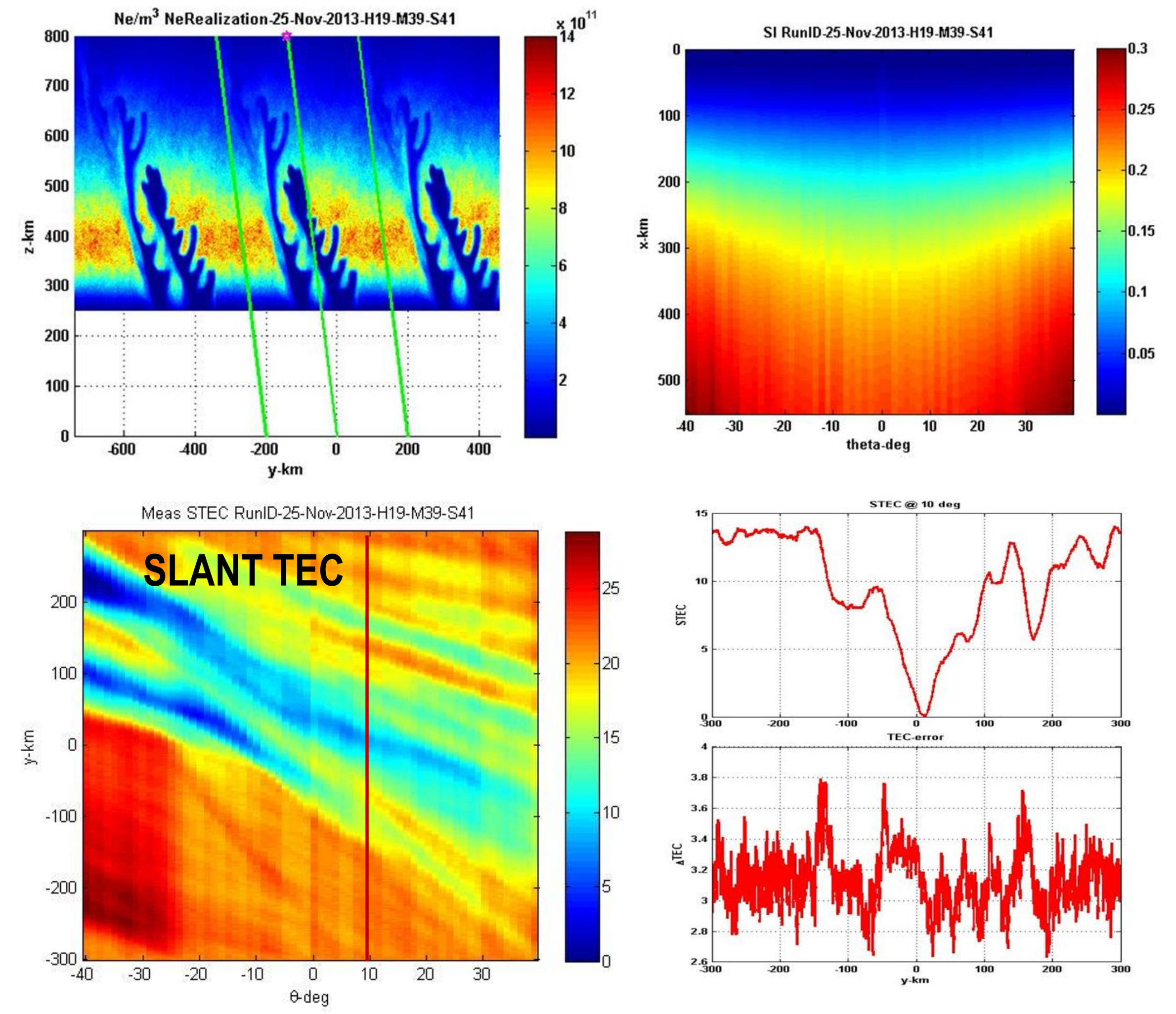
$$k\delta n(\boldsymbol{\rho}) = -2\pi K N_e(\boldsymbol{\rho})/f \text{ radians/Hz/m}$$

$$TEC = -f\bar{\phi}/(2\pi K)$$

PROPAGATION

$$\frac{\partial \psi_{\mathbf{k}}(x,\varsigma)(x,\boldsymbol{\rho})}{\partial x} = \underbrace{ik(\cos\theta-\Theta_k)\psi_{\mathbf{k}}(x,\varsigma)(x,\boldsymbol{\rho})} + ik\sec\theta\delta n(x,\boldsymbol{\rho}-\tan\theta\hat{\mathbf{a}}_{k_T}x)\psi_{\mathbf{k}}(x,\boldsymbol{\rho})(x,\boldsymbol{\rho})$$

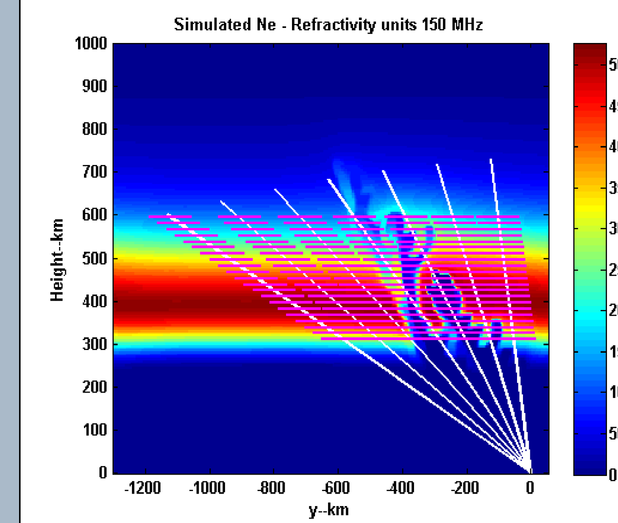
$$(\cos\theta-\Theta_k)\psi_{\mathbf{k}}(x,\boldsymbol{\rho}) = \iint \hat{\psi}_{\mathbf{k}}(0;\boldsymbol{\kappa})\exp\{i(k\cos\theta - g(\boldsymbol{\kappa}+\mathbf{k}_T)+\tan\theta\hat{\mathbf{a}}_{k_T}\cdot\boldsymbol{\kappa})x\} \times \exp\{i\boldsymbol{\kappa}\cdot\boldsymbol{\rho}\}\frac{d\boldsymbol{\kappa}}{(2\pi)^2}$$



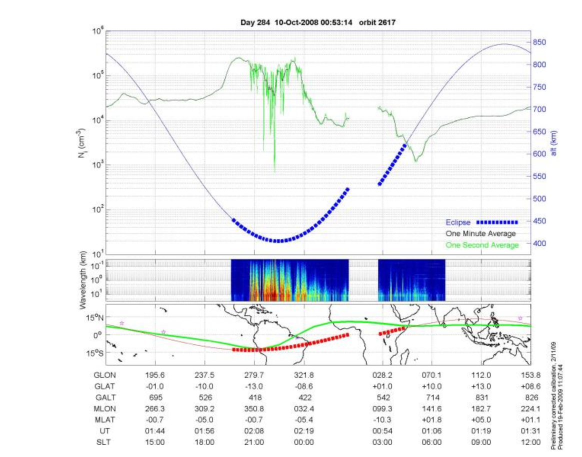
Open Issues for Future Research

- It is very difficult to isolate refractive deviations from the tomographic ideal. The fine structure generally associated with scintillation is noise like.
- The ramifications of highly field-aligned structure remains to be investigated. A fully three-dimensional model is being pursued by Kshitija Despande [SA24A-07].
- The practical limits of Tomographic reconstruction have yet to be determined. The medium itself generates stochastic anisotropic structure
  - Configuration space structure models that constrain stochastic variation to the cross-field direction are needed to generate realistic three-dimensional structure
  - The spatial distribution of intermediate scale structure and its characteristics are not sufficiently well defined to generate definitive model inputs

One Part of the Model Challenge
Background + Structure

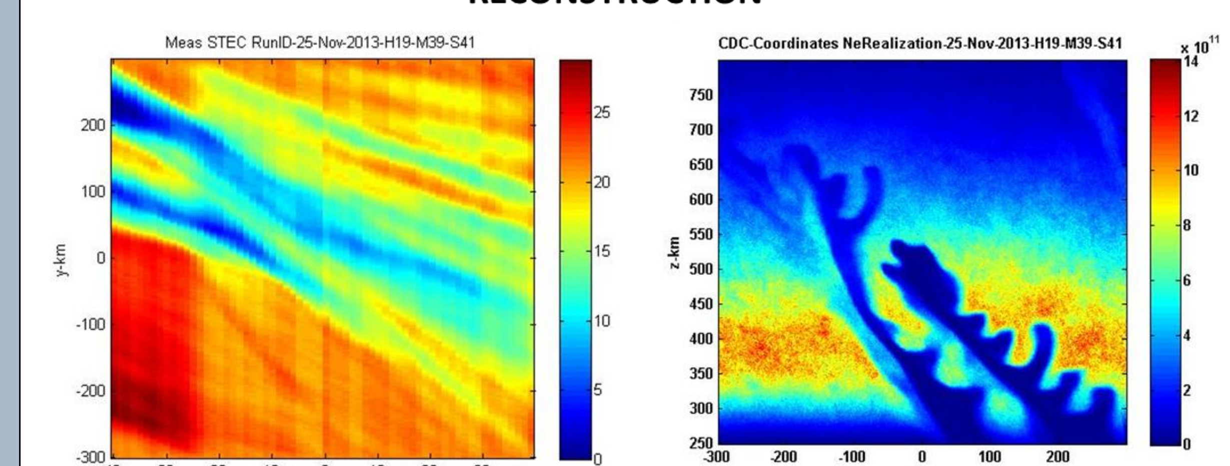


Physics-based models reproduce the 3D large-scale dynamic structure evolution to kilometer scales.



Satellite probes produce 1D scans that subtend ESF scales from hundreds of kilometers to meters.

RECONSTRUCTION



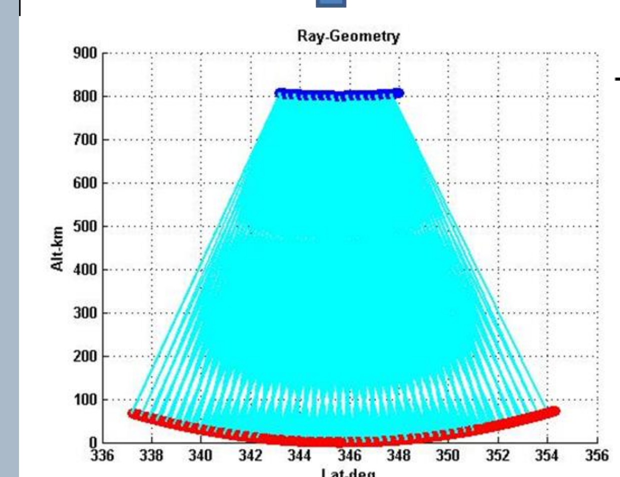
$$\phi_k^j = -(2\pi K/f)\sec\theta\int_0^L \delta N(\eta, k\Delta y - \eta\tan\theta_j)d\eta$$

$$\delta n(x,y) = \frac{1}{NM}\sum_{n=-N/2}^{N/2-1}\sum_{m=-M/2}^{M/2-1}\widehat{\delta n}_{nm}\exp\{i(n\Delta\kappa_x x + m\Delta\kappa_y y)\}$$

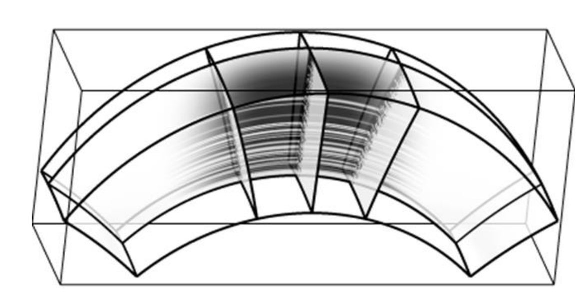
$$\hat{\phi}(m\Delta\kappa_y,\theta) = \left[\frac{1}{N}\sum_{n=-N/2}^{N/2-1}\Phi(n, m(\Delta\kappa_y/\Delta\kappa_x)\tan\theta)\widehat{\delta n}_{nm}\right]$$

$$\Phi_N(n,m\gamma) = \frac{2\pi\sec\theta L}{\lambda}\frac{\exp\{2\pi i(n-m\gamma)\}-1}{N\exp\{2\pi i(n-m\gamma)/N\}-1}$$

Horizontal DFT => Vertical DFT



32 Stations~42 km apart



$$\delta N(\zeta_\parallel,\boldsymbol{\zeta}_\perp)/N_0 = \frac{1}{N_s}\sum_k p_\parallel(\zeta_\parallel - \zeta_\parallel(\mathbf{r}_k))F_k p_\perp(|\boldsymbol{\zeta}_\perp - \boldsymbol{\zeta}_\perp(\mathbf{r}_k)|/\sigma_k)$$

$$\delta N(\zeta_\parallel;\boldsymbol{\kappa}_s)/N_0 \simeq \frac{1}{N_s}\sum_k F_k\sigma_k\hat{p}_\perp(\kappa_s\sigma_k)\exp\{-i\boldsymbol{\kappa}_s\cdot\boldsymbol{\xi}_\perp^k\}$$

$$\langle|\Delta N(\boldsymbol{\kappa}_s)/N_0|^2\rangle = \frac{1}{N_s}\sum_k\langle F_k^2\rangle\langle\sigma_k^2|\hat{p}_\perp(\kappa_s\sigma_k)|^2\rangle = \langle F^2\rangle\int_0^\infty p(\sigma)\sigma^2|\hat{p}_\perp(\kappa_s\sigma)|^2 d\sigma$$

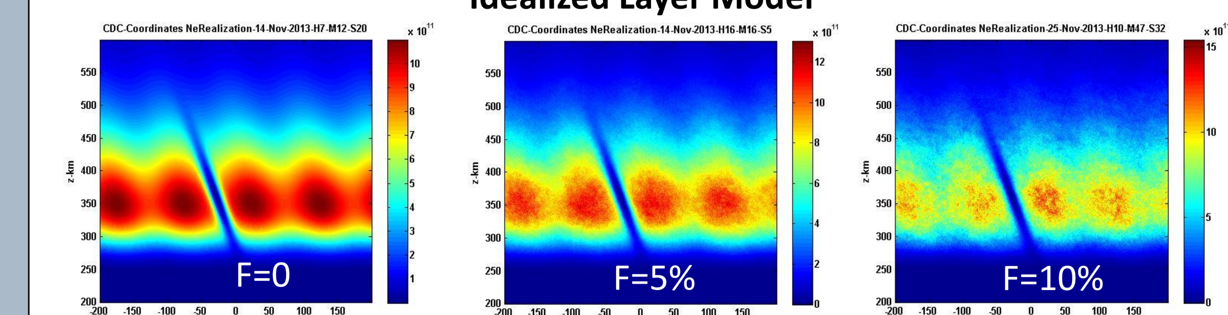
Power-law striation size distribution

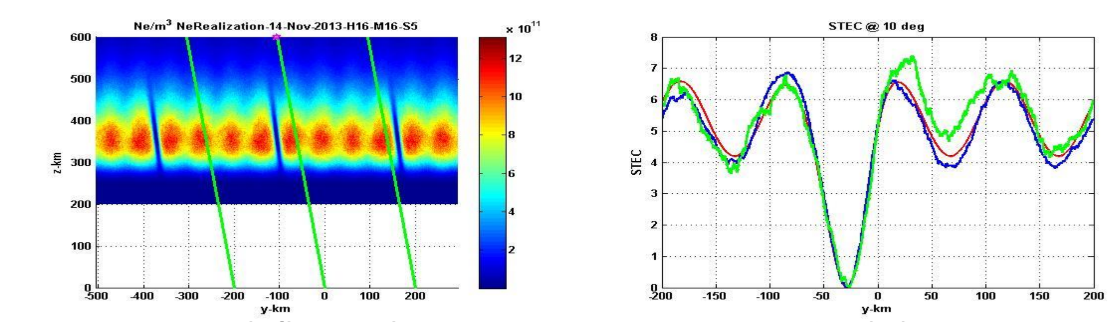
$$p(\sigma) = (2\nu-1)(1/\sigma_{\max})(\sigma/\sigma_{\max})^{2\nu-2} \text{ for } 0<\sigma<\sigma_{\max}$$

$$\langle|\Delta N(\boldsymbol{\kappa}_s)/N_0|^2\rangle = \langle F^2\rangle\underbrace{\left[\frac{2\nu-1}{\sigma_{\max}^{2\nu-1}}\int_0^{\kappa_s\sigma_{\max}} y^{2\nu}|\hat{p}_\perp(y)|^2 dy\right]}_{\text{Outer Scale}}\kappa_s^{-(2\nu+1)}$$

Inverse power-law variation

Two-Dimensional Realizations
Idealized Layer Model





Periodic extension to accommodate CDC coordinates

TEC variation Stochastic structure alone

Singular Value Decomposition

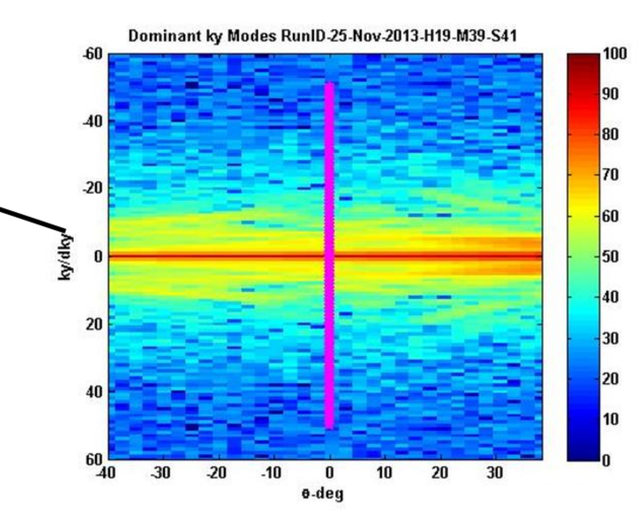
$$\hat{\boldsymbol{\phi}}_m = \Phi_m\widehat{\boldsymbol{\delta n}}_m$$

$$\Phi_m = USV'$$

$$S_n^{-1} = S(n)/(S^2(n)-\epsilon^2)$$

$$\Phi_m^{-1} = VS'U'$$

$$\widehat{\boldsymbol{\delta n}}_m = \Phi_m^{-1}\hat{\boldsymbol{\phi}}_m$$



2D DFT