Phase Screen Model Revisited

Chuck Rino

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Abstract

In a companion review *ScintillationTheoryRevisited* we showed that the forward propagation equation (FPE) can be applied to proplems involving propagation in a continuous inhomogeneous media as long as long as the propagation paths are near line-of-sight. Formally, the spectral density function (SDF) of the complex field must be confined to a narrow range of propagation angles. The free-space propagator can be replaced by a parabolic approximation, which effectively replaces propagation distance with the Fresnel scale. The resulting parabolic wave equation (PWE) can be evaluated with an efficient split-step procedure refered to as the multiple-phase-screen (MPS) method.

The scintillation theory summarized in *The Theory of Scintillation...* is a statistical theory that characterizes propagation in transparent random media. Although the development was initially introduced as a vector theory, it was found that an unmodified FPE structure cannot be extended to bi-refringent media, or scalar media. Simulations of wide-angle propagation in scalar media were demonstrated in *ScintillationTheoryRevisited*.

This report summarizes extended phase-screen simulations that demonstrate a new back-propagation application for extracting an estimate of path-integrated phase that supports irregularity parameter estimation (IPE).

1 Introduction

Scintillation, as characterized in the *The Theory of Scintillation ...*, is a complex narrow-band modulation imparted to propagating electromagnetic (EM) fields (signals) that have traversed the ionosphere. The more general phenomenon is EM propagation in continuously inhomogeneous media. The inhomogeneity subtends a broad range of structure scales with no definitive transition between systematic and stochastic variation. The formal connection between EM field observables and ionospheric electron density structure involves path integration of the electron density structure converted to phase.

In *Phase Screen Theory Revisited* we argued that propagation in extended disturbed ionospheric regions can be characterized by multiple phase screen (MPS) simulations based on a forward propagation equation (FPE). However, the constrained range of component plane-wave propagation angles effectively restricts applications to near-line-of-sight propagation from VHF (100 MHz and above) to S-Band (2 MHz and below), which is the range of the statistical theory of scintillation. Over the past several decades the statistical theory, which related observable statistical field moments to the SDF of the refractive index, has been pursued vigorously. Algorithmic results have been constructed for the complex field mutual coherence function (MCF) and the equivalent spectral-domain complex field SDF. Algorithmic results have also been constructed for the two-frequency intensity autocorrelation function (ACF). The algorithmic forms include the intensity SDF. In all cases combinations of the refractive index structure function characterize the media. The refractive index is a frequency-dependent scaling of the phase structure function. The basic results are summarize in Chapter 3 of [1]. A summary of the algorithmic implementations of applications to extended media can be found in [2].

Because of the complexity of implementing the algorithmic results, diagnostic applications were largely limited to special studies or weak-scatter and asymptotic limits. However, simulations were being used to validate theoretical results. The fact that theoretical results can be validated with operations applied to simulated realizations led to the possibility of eliminated the need for diagnostic analysis altogether. For example, FPE realizations are used exclusively for validating the localization of disturbances along occultation paths using back propagation [3].

Two recently published papers, [4] and [5], describe new scintillation diagnostic operations that characterize ionospheric structure. MPS simulations are used to validate the procedures. Although strictly a mathematical abstraction, the phase-screen model has been used extensively for constructing scintillation diagnostics. Realizations of complex scintillation are readily generated. Moreover formal expectation calculation of statistical observables admit theoretical prediction.

To summarize the essential elements a phase-screen realization of the complex scintillation field is defined by a phase realization $\phi(y)$ as follows:

$$\psi(x,y) = \exp\{ik\phi(y)\}. \tag{1}$$

The following path integration connects $\phi(y)$ to the electron-density structure, $\Delta N_e(x,y)$:

$$\phi(y) = 2\pi K/f \int_0^L \Delta N_e(x,y)dx. \tag{2}$$

The scale factor

$$K = r_e c/(2\pi) \times 10^{16}, \tag{3}$$

where r_e is the classical electron density and c is the vacuum velocity of light, converts total electron content (TEC) to phase. The variable x is the propagation distance, which is zero at the location of the phase screen.

Two-dimensional free propagation over a distance Δx is defined by the operation

$$\psi(x \pm \Delta x, y) = \int \widehat{\psi}(\kappa_y; x) \left[\exp\{\mp i \left(\sqrt{1-(\kappa_y/k)^2} - 1\right)(k\rho_F)^2\}\right] \times \exp\{i\kappa_y y\} d\kappa_y / (2\pi), \quad (4)$$

where $k = 2\pi f_c/c$, $\rho_F = \sqrt{\Delta x/k}$ is the Fresnel scale, and

$$\widehat{\psi}(\kappa_y; x) = \int \psi(x, y) \exp\{-i\kappa_y y\} dy \quad (5)$$

is the y Fourier transform of the complex field. Defining the propagation step as $\Delta x = (k\rho_F)^2$ anticipates the narrow-propagation-angle constraint, $(\kappa/k)^2 << 1$. The phase modification of each Fourier component takes the simpler form $\exp\{\pm i (\kappa_y \rho_F)^2 / 2\}$, which implies identical scintillation at distances and frequencies that preserve ρ_F.

The imposed phase structure is characterized in expectation by the spectral density function

$$\Phi_\phi(\kappa_y) = C_p \left\{ \begin{array}{r} \kappa_y{}^{-p_1},\ \kappa_y \le \kappa_0 \\ (\kappa_0{}^{p_2-p_1})\kappa_y^{-p_2},\ \kappa_y > \kappa_0 \end{array} \right. . \quad (6)$$

Four structure parameters, C_p, p_1, p_2, and κ_0, and the Fresnel scale ρ_F completely define a phase-screen realization.

The phase-screen theory predicts the intensity SDF as a function of a universal strength parameter

$$U = C_P \left\{ \begin{array}{ll} \rho_F^{p_1-1} & \text{for } \mu_0 \le 1 \\ \rho_F^{p_2-1}\mu_0^{p_1-p_2} & \text{for } \mu_0 \le 1 \end{array} \right. , \quad (7)$$

where $\mu_0 = \kappa_0 \rho_F$. Figures 1 and 2 in [4] compare theoretical calculations of $S4$ as a function of turbulent strength C_p for two structure models defined by different p_1 and p_2 values. The results show that for a given ρ_F, C_p structure levels that produce the same $S4$ values differ by orders of magnitude. However, because the phase-screen model is self-contained, agreement between theoretical predictions and statistical measures provide only checks on sampling adequacy. The $S4$ bias caused by additive noise also can be predicted theoretically as a function of SNR ([4] Eq. 41). If an SNR estimate is obtained independently, the bias can be corrected subject to measurement uncertainties.

Finally, because free propagation is perfectly reversible the intensity scintillation can be removed completely. Additionally, the mutual coherence function can be used to estimate the phase structure function, which admits an analytic representation for the two-component model. All of these relations have been exploited for diagnostic analysis. However, the results are strictly applicable only for structure initiated by a phase screen. The second paper, [5], uses MPS to extend the results to more realistic propagation in an extended medium. Theoretical results for propagation in an extended medium are not available in tractable algorithmic forms. Consequently, we use MPS simulations as truth.

2 Extended Phase Screen Model

The extension of the phase screen model to propagation in an extended medium follows by direct computation from 2. Repeating Equation (6) in [5]

$$\Phi_\phi(\kappa_y) = (2\pi KL/f)^2 \int \frac{\sin^2(\kappa_x L/2)}{(\kappa_x L/2)^2} \Phi_{\Delta N_e}(\kappa_x, \kappa_y) \frac{d\kappa_x}{2\pi}. \tag{8}$$

If the structure decorrelates within L, which is the usual assumption,

$$\Phi_\phi(\kappa_y) = (2\pi K/f_c)^2 L\Phi_{\Delta N_e}(0, \kappa_y). \tag{9}$$

Equating $\Phi_{\Delta N_e}(0, \kappa_y)$ with 6 follows from how the three-dimensional electron density SDF is defined. The difference between κ_y and the normal to the propagation direction can be corrected after structure estimation. MPS simulations that use uncorrelated phase realizations are effectively generating realizations characterized by $\Phi_\phi(\kappa_y)$ with $L\Phi_{\Delta N_e}(0, \kappa_y)$ replaced by 6. Back propagation as described in Section 1 is applied to MPS realizations.

Figures 1 and 3 and Figures 2 and 4 in [5] summarize the results of applying irregularity parameter estimation (IPE) to the measured SDF of the phase recovered from back propagation applied to MPS realization of the pure phase-screen structure demonstrated in [4]. Where the estimated phase screen propagation distance is only a consistency check, when applied to a 50 km MPS simulations the propagation distance indicated in the upper left frames of Figures 2 and 4 identify the center of the disturbed region, independent of the structure analysis. Regarding structure analysis, the intensity variation of the back-propagated signal from an extended disturbed region need be reduced significantly. Moreover, the parameter estimates derived from the recovered phase are affected by residual diffraction. Parameter and propagation distance estimates from multiple MPS realizations are shown in Figures 5 and 6. Extreme propagation-distance estimates are indicative of back propagation failure.

3 Data Analysis

The following signal model, which is summarized in [4],

$$\begin{aligned} v_d(t) &= A_d(t) h(t; f_c) \exp\{2\pi i K \cdot TEC(t)/f_c + \phi_\xi(t)\} \\ &\quad \times \exp\{2\pi i f_c t \dot{r}/c)\} + \epsilon(t)/\sqrt{SNR_d}. \end{aligned} \tag{10}$$

can be applied to any diagnostic receiver, where f_c is the transmitted frequency, $\phi_\xi(t)$ represents local oscillator noise, $\dot{r}$ represents range rate, $\epsilon(t)$ is a zero mean uncorrelated noise process. The complex scintillation modulation is $h(t; f_c)$. The amplitude variation $A_d(t)$ all processes that affect signal amplitude. The term $1/\sqrt{SNR_d}$ represents the noise amplitude.

3.1 Preprocessing

Preprocessing operations, which are applied to sampled intensity and phase data, $I(n\Delta t) = |v_d(n\Delta t)|^2$ and $\phi(n\Delta t)$, are designed to isolate the scintillation modulation, $h(t; f_c)$, the total electron content, $TEC(t)$, and the geometric Doppler $2\pi i f_c \dot{r}/c$. In a well-designed system the sample rate is high enough to allow estimation of SNR_d. It is convenient to scale $I(n\Delta t)$ to SNR units.

3.1.1 Detrending

Detrending starts with an estimate of background intensity variation. Determining the largest interval over which background variations are occurring determines the smallest frequency scale that can be isolated. Wavelet based *de noising* is effective and efficient. If $\overline{I}(n\Delta t)$ represents the trend estimate, (10) normalized to $\sqrt{\overline{I}(n\Delta t)}$ can be rewritten as

$$\begin{aligned}\overline{v}_d(n\Delta t) \quad \simeq \quad & \sqrt{1+1/SNR}\psi(n\Delta t; f_c)\exp\{2\pi i K\cdot TEC(n\Delta t)/f_c + \phi_\xi(n\Delta t)\} \\ & \times \exp\{2\pi i f_c n\Delta t\dot{r}/c)\}. \qquad (11)\end{aligned}$$

The average intensity of $\overline{v}_d(t)$ is $1+1/SNR$ over the detrend interval T. Figure 7 in [5] shows examples of intensity data converted to SNR units with the trends overlaid.

Applying a complex field phase unwrapping algorithm to $\overline{v}_d(n\Delta t)$ generates the unmodified complex signal phase

$$\phi_v(n\Delta t) \simeq 2\pi i f_c n\Delta t\dot{r}/c + 2\pi i K\cdot TEC(n\Delta t)/f_c + \phi_s(n\Delta t; f_c) + \phi_\xi(n\Delta t). \quad (12)$$

The contributions have been ordered by peak magnitude contribution. For TEC estimation geometry-free phase combinations remove the leading term leaving frequency-scaled $TEC(n\Delta t)$ and phase scintillation and oscillator error terms. The oscillator noise is usually negligibly small. The scintillation term is an error for TEC estimation. However, for diagnostic measurement $\phi_s(n\Delta t; f_c)$ is the desired result.

3.1.2 Complex Field Extraction

For structure analysis, a polynomial detrend operation removes the geometric Doppler. There is a TEC residual indistinguishable from the scintillation phase structure, which can be negligibly small. Segmentation identifies T sec data segments with T adjusted to an integral number of samples. A linear trend is removed from the segment phase to avoid discontinuities. To test data homogeneity $S4$ estimates are made over T intervals offset by $T/2$. Figure 8 in [5] shows the $S4$ estimates for the $L1$ and $L2$ frequencies shown in Figure 7.

3.2 Back Propagation

BP processing is initiated with a starting propagation distance. However, the complex field must be translated to distance, $y = v_{\text{eff}}t$. The segmentation interval T and Δt determine the number of samples per segment, N. For DFT processing it is covenient to chose an efficient DFT number, n_{fft}, for spectral analysis. It is also convenient to choose v_{eff} so that the Nyquist frequency corresponds to the smallest propagating wavenumber:

$$v_{\mathrm{eff}} = \pi n_{\mathrm{fft}} k/2. \tag{13}$$

Minimizing the x-dependent intensity variance of the complex field defined by (4) generates a new complex field and propagation distance. If minimization fails a flag is set and the BP field using the starting propagation distance is reported. Standard phase unwrapping recovers the BP phase.

3.3 Irregularity Parameter Estimation

A periodogram estimate of the BP phase SDF is used for IPE estimation of the structure parameters. An IPE search is initiated with a two-component SDF constructed with linear fits to each half of the log-log SDF. However, the IPE search needs to be confined to the SDF structure above the noise floor. Otherwise the noise floor will confuse the two-component model. We found that applying IPE over the full frequency range identifies the noise floor. A second constrained IPE applicaton recovers the desired scintillation structure. lthough the measured parameters maximize likelihood they are subject to bias and correlation [6].

4 Standard Summary Parameters

The standard scintillation parameters $S4$, σ_ϕ, and $ROTI$ estimates can be generated as follows:

$$S4 = \sqrt{\sum_{n=1}^{ny} I_n^2 / \left(\sum_{n=1}^{ny} I_n\right)^2 - 1} \tag{14}$$

$$\sigma_\phi = \sqrt{\frac{1}{ny}\sum_{n=1}^{ny} \phi_n^2 - \left(\frac{1}{ny}\sum_{n=1}^{ny} \phi_n\right)^2} \tag{15}$$

$$ROTI(M) = \sqrt{\frac{1}{[M,m]}\sum \left(\phi([M,m]+M) - \phi([M,m])\right)^2} \tag{16}$$

The notation $[M,m]$ indicates the set of integers spaced by M that span a segment of length ny. For power-law environments only $S4$ and $ROTI(M)$ converge to definitive values. Rms phase, σ_ϕ, depends on the interval over which it is measured.

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